Friday, October 16, 2015

p. 521: 19, 21, 22, 25, 27, 29, 50, 51, 55, 61, 67, 69

Problem 19

Problem. Find the indefinite integral $\int \frac{xe^{2x}}{(2x+1)^2}\,dx$.

Solution. Let $u = xe^{2x}$ and $dv = \frac{dx}{(2x+1)^2}$. Then $du = (2x+1)e^{2x}\,dx$ (use the Product Rule) and $v = -\frac{1}{2(2x+1)}$.

$$\begin{aligned}\int \frac{xe^{2x}}{(2x+1)^2}\,dx &= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{1}{2(2x+1)} \cdot (2x+1)e^{2x}\,dx \\ &= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{2}\int e^{2x}\,dx \\ &= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C.\end{aligned}$$

Problem 21

Problem. Find the indefinite integral $\int x\sqrt{x-5}\,dx$.

Solution. Let $u = x$ and $dv = \sqrt{x-5}$. Then $du = dx$ and $v = \frac{2}{3}(x-5)^{3/2}$.

$$\begin{aligned}\int x\sqrt{x-5}\,dx &= \frac{2}{3}x(x-5)^{3/2} - \frac{2}{3}\int ((x-5)^{3/2}\,dx) \\ &= \frac{2}{3}x(x-5)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(x-5)^{5/2} \\ &= \frac{2}{15}(3x+10)(x-5)^{3/2} + C.\end{aligned}$$

Problem 22

Problem. Find the indefinite integral $\int \frac{x}{\sqrt{6x+1}}\,dx$.

Solution. Let $u = x$ and $dv = \dfrac{dx}{\sqrt{6x+1}}$. Then $du = dx$ and $v = \frac{1}{3}(6x+1)^{1/2}$.

$$\begin{aligned}\int \frac{x}{\sqrt{6x+1}}\,dx &= \frac{1}{3}x(6x+1)^{1/2} - \frac{1}{3}\int (6x+1)^{1/2}\,dx \\ &= \frac{1}{3}x(6x+1)^{1/2} - \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{6}(6x+1)^{3/2} + C \\ &= \frac{1}{3}x(6x+1)^{1/2} - \frac{1}{27}(6x+1)^{3/2} + C \\ &= \frac{1}{27}(3x-1)\sqrt{6x+1} + C.\end{aligned}$$

Problem 25

Problem. Find the indefinite integral $\displaystyle\int x^3 \sin x\,dx$.

Solution. Hey, let's use the tabular method.

Sign	u	dv
$+$	x^3	$\sin x$
$-$	$3x^2$	$-\cos x$
$+$	$6x$	$-\sin x$
$-$	6	$\cos x$
$+$	0	$\sin x$

Then

$$\int x^3 \sin x\,dx = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C.$$

Problem 27

Problem. Find the indefinite integral $\displaystyle\int \arctan x\,dx$.

Solution. This problem is similar to integrating $\ln x$. The only choices are $u = 1$, $dv = \arctan x\,dx$ (pointless) and $u = \arctan x$, $dv = dx$. So let's try the one that is not pointless. Then $du = \dfrac{dx}{x^2+1}$ and $v = x$.

$$\begin{aligned}\int \arctan x\,dx &= x\arctan x - \int \frac{x}{x^2+1}\,dx \\ &= x\arctan x - \frac{1}{2}\ln(x^2+1) + C.\end{aligned}$$

Problem 29

Problem. Find the indefinite integral $\int e^{-3x} \sin 5x \, dx$.

Solution. Let $u = e^{-3x}$ and $dv = \sin 5x \, dx$. Then $du = -3e^{-3x} \, dx$ and $v = -\frac{1}{5} \cos 5x$.

$$\int e^{-3x} \sin 5x \, dx = -\frac{1}{5} e^{-3x} \cos 5x - \frac{3}{5} \int e^{-3x} \cos 5x \, dx.$$

Do it again. Let $u = e^{-3x}$ and $dv = \cos 5x \, dx$. Then $du = -3e^{-3x} \, dx$ and $v = \frac{1}{5} \sin 5x$.

$$\begin{aligned} \int e^{-3x} \sin 5x \, dx &= -\frac{1}{5} e^{-3x} \cos 5x - \frac{3}{5} \left(\frac{1}{5} e^{-3x} \sin 5x + \frac{3}{5} \int e^{-3x} \sin 5x \, dx \right) \\ &= -\frac{1}{5} e^{-3x} \cos 5x - \frac{3}{25} e^{-3x} \sin 5x - \frac{9}{25} \int e^{-3x} \sin 5x \, dx. \end{aligned}$$

Therefore,

$$\frac{34}{25} \int e^{-3x} \sin 5x \, dx = -\frac{1}{5} e^{-3x} \cos 5x - \frac{3}{25} e^{-3x} \sin 5x.$$

and so

$$\int e^{-3x} \sin 5x \, dx = -\frac{5}{34} e^{-3x} \cos 5x - \frac{3}{34} e^{-3x} \sin 5x + C.$$

Problem 50

Problem. Use the tabular method to find the integral $\int x^3 e^{-2x} \, dx$.

Solution. The table:

Sign	u	dv
$+$	x^3	e^{-2x}
$-$	$3x^2$	$-\frac{1}{2}e^{-2x}$
$+$	$6x$	$\frac{1}{4}e^{-2x}$
$-$	6	$-\frac{1}{8}e^{-2x}$
$+$	0	$\frac{1}{16}e^{-2x}$

Then

$$\begin{aligned} \int x^3 e^{-2x} \, dx &= -\frac{1}{2} x^3 e^{-2x} - \frac{3}{4} x^2 e^{-2x} - \frac{3}{4} x e^{-2x} - \frac{3}{8} e^{-2x} \\ &= \left(-\frac{1}{2} x^3 - \frac{3}{4} x^2 - \frac{3}{4} x - \frac{3}{8} \right) e^{-2x} \\ &= -\frac{1}{8} (4x^3 + 6x^2 + 6x + 3) e^{-2x} + C. \end{aligned}$$

Problem 51

Problem. Use the tabular method to find the integral $\int x^3 \sin x \, dx$.

Solution. We already did this in Exercise 25.

Problem 55

Problem. Find the indefinite integral $\int \sin \sqrt{x} \, dx$ by using substitution followed by integration by parts.

Solution. We will use integration by parts in a few moments, so let's use t for the substitution. Let $t = \sqrt{x}$, or, equivalently, $x = t^2$. Then $dx = 2t \, dt$. We get

$$\begin{aligned} \int \sin \sqrt{x} \, dx &= \int \sin t \cdot 2t \, dt \\ &= 2 \int t \sin t \, dt. \end{aligned}$$

Now let $u = t$ and $dv = \sin t \, dt$. Then $du = dt$ and $v = -\cos t$.

$$\begin{aligned} 2 \int \sin \sqrt{x} \, dx &= -2t \cos t + 2 \int \cos t \, dt \\ &= -2t \cos t + 2 \sin t + C \\ &= -2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C. \end{aligned}$$

Problem 61

Problem. State whether you would use integration by parts to evaluate each integral. If so, identify what you would use for u and dv.

(a) $\int \frac{\ln x}{x} \, dx$

(b) $\int x \ln x \, dx$

(c) $\int x^2 e^{-3x} \, dx$

(d) $\int 2x e^{x^2} \, dx$

(e) $\int \frac{x}{\sqrt{x+1}}\, dx$

(f) $\int \frac{x}{\sqrt{x^2+1}}\, dx$

Solution. (a) No need for integration by parts. This can be done with the simple substitution $u = \ln x$, $du = \frac{dx}{x}$.

(b) Use integration by parts. Let $u = \ln x$ and $dv = x\, dx$.

(c) Use integration by parts twice. To get started, let $u = x^2$ and $dv = e^{-3x}\, dx$.

(d) No need for integration by parts. This can be done with the simple substitution $u = x^2$, $du = 2x\, dx$.

(e) This is very similar to Exercise 22. Use integration by parts. Let $u = x$ and $dv = (x+1)^{-1/2}\, dx$.

(f) No need for integration by parts. This can be done with the simple substitution $u = x^2 + 1$, $du = 2x\, dx$.

Problem 67

Problem. Use integration by parts to prove the formula

$$\int x^n \sin x\, dx = -x^n \cos x + n \int x^{n-1} \cos x\, dx$$

Solution. Use integration by parts one time. Let $u = x^n$ and $dv = \sin x\, dx$. Then $du = nx^{n-1}\, dx$ and $v = -\cos x$.

$$\int x^n \sin x\, dx = -x^n \cos x + n \int x^{n-1} \cos x\, dx.$$

Problem 69

Problem. Use integration by parts to prove the formula

$$\int x^n \ln x\, dx = \frac{x^{n+1}}{(n+1)^2}\left[-1 + (n+1)\ln x\right] + C$$

Solution. Use integration by parts one time. Let $u = \ln x$ and $dv = x^n\ dx$. Then $du = \frac{1}{x}\ dx$ and $v = \frac{1}{n+1}x^{n+1}$.

$$\begin{aligned}
\int x^n \ln x\ dx &= \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{n+1}\int x^{n+1}\cdot\frac{1}{x}\ dx \\
&= \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{n+1}\int x^n\ dx \\
&= \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{n+1}\cdot\frac{1}{n+1}x^{n+1} + C \\
&= \frac{1}{(n+1)^2}\big((n+1)x^{n+1}\ln x - x^{n+1}\big) + C \\
&= \frac{x^{n+1}}{(n+1)^2}\left((n+1)\ln x - 1\right) + C \\
&= \frac{x^{n+1}}{(n+1)^2}\left(-1 + (n+1)\ln x\right) + C.
\end{aligned}$$